HEAVY MAJORANA NEUTRINO DECAYS AND CP-PARITY VIOLATION

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Abstract

In the framework of the left-right symmetric model the CP-parity violation has been studied in heavy Majorana neutrino decays $N \to e^{\mp}\mu^{\pm}n$. The numerical estimates of CP-asymmetry for different masses of neutrino and $W_R^{\pm}$-boson were obtained. The possibility to detect this phenomenon at high energy colliders is mentioned.

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Recently the heavy Majorana neutrino production and their further decays in e^+e^-, ep, pp -collisions were actively studied (see [1, 2] and references therein). Heavy Majorana neutrinos appear in many theories beyond the Standard Model (massive Majorana particles, gluino, neutralino, gravitino, appear also in supergravity theories). Their introduction helps to solve such important problems as the smallness of the left-handed neutrino mass (see the Appendix A for the neutrino mass matrix), baryonic asymmetry of universe and etc [3, 4, 5, 6]. On the other hand, the introduction of Majorana neutrinos leads to some new phenomena such as the double neutrinoless β-decay and other processes with lepton number violation. Recently, in some works [2, 7] there have been shown that CP-parity violation can be induced in the decays of Higgs bosons ($H^o \to \bar{t}t, W^+W^-, Z^oZ^o$) by heavy Majorana neutrino loops.

The aim of this work is the investigation of CP-parity violation in heavy Majorana neutrino decays $N \to e^{\mp}\mu^{\pm}n$. It is well known that the existence of CP-violation requires the presence of two phases. In our case the CP violation arises due to the interference of two diagramms (see Fig. 1). The resulting CP asymmetry is proportional to the phases of Kobayashi-Maskawa like mixing matrix and the imaginary part of one of the diagramms of Fig. 1, which is connected with nonzero width of $W_R^{\pm}$- boson. It should be mentioned that this phenomenon, is absent in the case of the Dirac neutrino decay, because in this case the decay is described by only one diagram. It should be noted that CP-parity violation was investigated also in $N \to e^+e^-n$ decays [8]. It appeared due to the interference of two W-boson diagrams with the third diagram, also contributing to this process and containing nondiagonal Z^oNn vertex, absent in cases with Dirac neutrino. In case of the $N \to e^+e^-n$ decay the interference of both W-bosonic diagrams with each other did not lead to CP-parity violation [8].

We are working in the framework of left-right symmetric model, however our numerical results are applicable also to the models without $W_R^{\pm}$ e.g. to the Standard Model with right neutrinos [1]. In this case $N \to e^{\mp}\mu^{\pm}n$ decays take place due to the exchange of standard $W_L^{\pm}$-bosons.

The total widths of processes $N \to e^{\pm}\mu^{\mp}n$ - $\Gamma \equiv \Gamma(N \to e^-\mu^+n)$ and $\overline{\Gamma} \equiv \Gamma(N \to e^+\mu^-n)$ described by Fig. 1 are equal to each other. However, the differential widths, as it will be seen below, can be different. To compare the differential widths of these decays let us consider the partialy integrated decay

widths $\Gamma_{y_1>z}$, $\bar{\Gamma}_{y_2>z}$, where $y_{1,2}=\frac{2\epsilon_{1,2}}{m}$, $\epsilon_{1,2}$ are energies of $e^-,\mu^+(\mu^-,e^+)$ in $N\to e^-\mu^+n,(N\to e^+\mu^-n)$ processes, m is mass of neutrino N. Thus $\Gamma_{y_1>z}$ is proportional to the number of events $e^-\mu^+n$ in the decay $N\to e^-\mu^+n$ with electron energy $\epsilon_{e_1}>\epsilon_o$ and $\bar{\Gamma}_{y_2>z}$ is proportional to the numbers of events in the decay $N\to e^+\mu^-n$ with positron energy $\epsilon_{e_2}>\epsilon_o$, ($z=\frac{2\epsilon_o}{m}$).

Deriving Kobayashi-Maskawa (KM) matrix elements in the $N\to e^{\mp}\mu^{\pm}n$ amplitude processes:

$$\begin{aligned} M &= V_{Ne}V^*_{n\mu}M_1 - V^*_{N\mu}V_{ne}M_2 \\ \bar{M} &= V_{N\mu}V^*_{ne}M_1 - V^*_{Ne}V_{n\mu}M_2 \end{aligned} \tag{1}$$

after some simple calculations ($M_i,\overline{M_i}$ are determined in formulae (5), (6)) we have:

$$\Gamma_{y_1>z}-\bar{\Gamma}_{y_2>z}=2\mathrm{Im}\,(V_{Ne}V^*_{n\mu}V_{N\mu}V^*_{ne})\frac{1}{4m}\int\limits_{y_1>z}\mathrm{Im}\,(M_1M_2^+)\,d\Phi \tag{2}$$

Here $d\Phi$ is the differential three-particle phase space.

For $z=0$ formula (2) results to the total widths difference, which is equal to zero in accordance with formula (B5) as we have mentioned above. From (2) one can see that the effect exists only if the factor $\mathrm{Im}\,(V_{Ne}V^*_{n\mu}V_{N\mu}V^*_{ne})$ is different from zero. Indeed, all four elements of KM matrix in interaction Wen (formula (A7)), included in this factor are at the verteces of rectangles (formula (3) below), by the phase transformations of fermionic fields one can eliminate the phase only of one line and one column in V matrix (in formula (3) they are marked out by solid lines). Thus, at least one of above mentioned KM matrix elements may have imaginary part.

$$V=\left(\begin{array}{c|c|ccc} \cdots & \cdot & \cdots & \cdot & \cdots \\ \hline \cdots & V_{Ne} & \cdots & V_{ne} & \cdots \\ \hline \cdots & \cdot & \cdots & \cdot & \cdots \\ \cdots & V_{N\mu} & \cdots & V_{n\mu} & \cdots \\ \cdots & \cdot & \cdots & \cdot & \cdots \end{array}\right) \tag{3}$$

As it is seen from formula (2), for the existing of CP-parity violation the phase difference between amplitudes M_1 and M_2 ($\mathrm{Im}M_1M_2^+\neq 0$) is also necessary.

In [9, 10, 11] CP-parity violation due the interference of the imaginary part of tree diagram with a real part of loop diagram in t-quark decay $t\to$

$d_i\bar{d}_j u$ was studied (see also [12], where CP violation appeared due to the t-quark nonzero width). In our case the effect appears due to the fact that for one of diagrams $W_R^{\pm}$ boson is on mass shell (as a result the imaginary part of the W- boson propagator occurs) and is virtual on the second diagram and vise versa.

It must be noted that, the CP- violation can arise also due to the interference of diagramms Fig. 2 (see also [13]) with the diagramms Fig. 1. Below we will show (see formula (18)) that their contribution is relatively small.

Thus, we consider the range of masses:

$$m > m_R > m_n \tag{4}$$

where m_n is the mass of the neutrino n. It should be noted [9], that we call diagrams 1 and 2 as of a tree type conditionally; each of them is the result of summation of infinite number of diagrams with quark and lepton loops. The finite width in the $W_R^{\pm}$-boson propagator arises due to the imaginary parts of these loops (see formula (5), (6) below).

The amplitudes M_1, M_2 may be written as follows:

$$M_1 = \frac{g^2}{2}\bar{u}(k_1)\gamma_\mu P_R v(k)\bar{u}(k_3)\gamma_\nu P_R v(k_2)\frac{1}{m^2}\frac{1}{1-y_1-r_R^2+i\gamma}\times \\ \times\left[g_{\mu\nu}-\frac{(k-k_1)_\mu(k-k_1)_\nu}{m_R^2}\right] \tag{5}$$

$$M_2 = \frac{g^2}{2}\bar{u}(k)\gamma_\mu P_R v(k_2)\bar{u}(k_1)\gamma_\nu P_R v(k_3)\frac{1}{m^2}\frac{1}{1-y_2-r_R^2+i\gamma}\times \\ \times\left[g_{\mu\nu}-\frac{(k-k_2)_\mu(k-k_2)_\nu}{m_R^2}\right] \tag{6}$$

Here $r = \frac{m_n}{m}$, $r_R = \frac{m_R}{m}, \gamma = r_R\frac{\Gamma_R}{m}$, symbols are introduced, Γ_R- is the width of $W_R^{\pm}$-boson. Let us introduce the following definition of CP-asymmetry:

$$A_{CP} = \frac{\Gamma_{y_1>z}-\bar{\Gamma}_{y_2>z}-\Gamma_{y_1<z}+\bar{\Gamma}_{y_2<z}}{\Gamma_{y_1>z}+\bar{\Gamma}_{y_2>z}+\Gamma_{y_1<z}+\bar{\Gamma}_{y_2<z}} = \frac{\Gamma_{y_1>z}-\bar{\Gamma}_{y_2>z}-\Gamma_{y_1>z}+\bar{\Gamma}_{y_2>z}}{2\Gamma}. \tag{7}$$

While deriving formula (6) the obvious identities were used:

$$\begin{aligned}\Gamma_{y_1>z} + \Gamma_{y_1<z} &= \Gamma \\ \bar{\Gamma}_{y_2>z} + \bar{\Gamma}_{y_2<z} &= \Gamma.\end{aligned} \tag{8}$$

Thus, we have the difference of events $e^-\mu^+n$ with electron energies $\epsilon_{e^-} > \epsilon_o$ and $e^+\mu^-n$ with positron energy $\epsilon_{e^+} > \epsilon_o, z = \frac{2\epsilon_o}{m}$.

We can define more "symmetric" asymmetry

$$A_{CP} = \frac{\Gamma_{y_1>z} - \bar{\Gamma}_{y_2>z} - \Gamma_{y_1<z} + \bar{\Gamma}_{y_2<z} - (\Gamma_{y_2>z} - \bar{\Gamma}_{y_1>z} - \Gamma_{y_1<z} + \bar{\Gamma}_{y_2<z})}{4\Gamma} \tag{9}$$

Using the formulae (1,2,5,6) one can show, that (7) and (9) asymmetries are equal to each other.

As a result of our calculations (Appendix B), we have:

$$A_{CP} = vf(r, r_R, z), \tag{10}$$

where

$$v = \frac{\mathrm{Im}(V_{Ne}V^*_{n\mu}V_{N\mu}V^*_{ne})}{|V_{Ne}|^2|V_{n\mu}|^2 + |V_{N\mu}|^2|V_{ne}|^2} \tag{11}$$

It is obvious, that $v \leq 0.5$. The dependence of the function f on z at various r, r_R is shown on Figs 3-5. From Fig. 3-5 we see that the effect is maximal for some middle values of z.

In our calculations we assumed the main W_R-boson decay modes to be the decays in hadrons $W_R \to ud$ as well as $W_R \to ln$ and so

$$\Gamma_R = 3\Gamma_{ud} + \Gamma_{ln} \tag{12}$$

Let us estimate the possibility of the effect observation at high energy colliders. Using the formula for $W_R \to en$ decay widths [14]

$$B(W_R \to en) \approx 0.04|V_{Ne}|^2 \quad \left[2 + \frac{m_n^2}{m_R^2}\right](1 - \frac{m_n^2}{m_R^2}) \tag{13}$$

and also (B10) we have:

$$B(N \to ne^-\mu^+) = 0.02\left[|V_{Ne}|^2|V_{N\mu}|^2 + |V_{Ne}|^2|V_{n\mu}|^2\right] \quad \left[2 + \frac{m_n^2}{m_R^2}\right](1 - \frac{m_n^2}{m_R^2}) \tag{14}$$

Hence, the difference of event number is:

$$\Delta N_{CP} = R(B(N \to e^-\mu^+ n)_{\varepsilon_{e^-}>\varepsilon_0} - B(N \to e^+\mu^- n)_{\varepsilon_{e^+}>\varepsilon_0} \quad (15)$$
$$-B(N \to e^-\mu^+ n)_{\varepsilon_{e^-}<\varepsilon_0} - B(N \to e^+\mu^- n)_{\varepsilon_{e^+}<\varepsilon_0}) = 2RA_{CP}B(N \to e^+\mu^- n)$$

Here R is the number of heavy neutrinos produced in colliders, $R = 2\sigma \int \mathcal{L}dt$- for the production of the pair of Majorana neutrino, $R = \sigma \int \mathcal{L}dt$ - for production of the single Majorana neutrinos. Taking into account that $\sqrt{s} \gg m_R, m_N$ [1] for the diagrams with t-chanel exchange of W_R boson we have:

$$\sigma(e^+e^- \to NN) = |V_{Ne}|^4 \times 100\text{nbn} \left(\frac{m_L}{m_R}\right)^2 \quad (16)$$

From (16) one has for $m_R \sim 800GeV$ and integrated luminosites $\mathcal{L} = 3 \cdot 10^{41}s^{-1}$ number of neutrinos $R = 6 \cdot 10^5 |V_{Ne}|^4$. Taking into account that $f \sim 0.1 \div 0.5$ for wide range of masses and middle values of z one has:

$$\Delta N_{CP} = (2.5\times10^3 \div 1.25\times10^4) r \text{Im}(V_{Ne}V^*_{n\mu}V_{N\mu}V^*_{ne})|V_{Ne}|^4(1-\frac{m_n^2}{m_R^2})\left\{2 + \frac{m_n^2}{m_R^2}\right\} \quad (17)$$

The heavy neutrinos may also be produced at the peak of Z'- bosons. From [15] (p.496) we have, that when $m_{Z'} = 750GeV$ and $\mathcal{L} = 10^{34}cm^{-2}s^{-1}$, the number of neutrino $R \sim 3 \times 10^7$ (assuming $m_{Z'} \gg 2m_N$ and so $\Gamma(Z' \to NN) \sim \Gamma(Z' \to \mu^+\mu^-)$). So, in decays of Z' the ΔN_{CP} is increased at least by 2-3 orders.

It should be also noted, that our results can be also applicable to the models with heavy Majorana neutrino without $W_R^{\pm}$-bosons. However, in these models $N \to e^{\mp}\mu^{\pm}n$ processes occur due to the standard $W_L^{\pm}$- boson exchange, the interaction of which with N and n contains the additional smallness of order ξ in comparison with $W_R^{\pm}Ne$ interaction.

As mentioned above the considered effect of CP violation arises due to the existence of nonzero width of W_R- boson, which is the result of summation of infinit number quark and leptons loop diagramms. That is why we must estimate the contribution of other loop diagramms. Let us consider the contribution of diagramms Fig. 2. The contribution of diagramm Fig. 2a are GIM supressed by factor $V_{Ne}V^*_{ne}\frac{m_l^2}{m_N^2}$ which arise from internal charged lepton lines. More considerable contribution comes from the interference of diagramm Fig. 2b with diagramms of Fig. 1. The CP -asymmetry due to

this contribution is of order

$$A_{CP} \sim \frac{v\alpha^2 r}{\sum B(W_R \to ln)} \sim 10^{-3} rv \tag{18}$$

(this estimate is true when m_n is not very close to m_R), where α is fine structure constants. As a result we see that for optimal z's —(Fig. 3-5) the contribution of the diagramm Fig. 2 is 100-10 times smaller of our result.

There exist the additional loop diagramms with Z-bosons and photon exchange which can contribute to the considered process. Their contribution is of the same order as diagramms of Fig. 2.

We plan also to study CP odd correlations in decays of charged leptons, particularly, in $\mu^{\pm} \to e^{\pm}\nu_i\nu_j$ decays, where μ-meson and/or electron are polarized. The work in this direction is presently in progress.

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APPENDIX A

Spontaneous symmetry breaking of $SU(2)_L \times SU(2)_R \times U(1)$ leads to a mass terms in the neutrino sector:

$$L_m = -\overline{\nu}_L m_D \nu_R - \overline{\nu}^c_R M \nu_R + h.c. \tag{A.1}$$

The Dirac (m_D) and Majorana (M) mass matrices are related to the standard doublet and right Higgs triplet vacuum expectation:

$$\begin{aligned} m_D &= f_\nu < \varphi > \\ M &= h < \chi > \end{aligned} \tag{A.2}$$

One can obtain the connection between the weak eigenstates ν_L and ν_R and the Majorana mass eigenstates ν, N as a power series in $\xi = \frac{m_D}{M}$ (assuming $\det M \gg \det m_D$) [1]:

$$\nu_L = P_L \nu + \xi P_L N + O(\xi^2) \tag{A.3}$$

$$\nu_R = P_R \nu + \xi^T P_R \nu + O(\xi^2) \tag{A.4}$$

where $P_{L,R} = (1 \pm \gamma_5)/2$. The masses of N and ν are

$$M_N = M + O(\frac{1}{M}) \tag{A.5}$$

$$m_\nu = -m_D \frac{1}{M} m_D^T + O(\frac{1}{M^3}) \tag{A.6}$$

Thus, requirement $< \varphi > \ll < \chi >$ naturally leads to large Majorana mass for right-handed neutrinos. Left-handed neutrinos are nearly massless. Above requirement provides also large $W_R^\pm$-masses. Interaction of the right-handed neutrinos with $W_R^\pm$-bosons has the following form:

$$L = \frac{g}{\sqrt{2}} \bar{l} \hat{W}_R P_R V N + h.c. \tag{A.7}$$

In (7) V is a Kobayashi-Maskawa type mixing matrix in the leptonic part of the charged current.

APPENDIX B

For width difference of $N \to e^{\pm}\mu^{\mp}n$ processes we have:

$$\Gamma_{y_1>z} - \overline{\Gamma}_{y_2>z} = \mathrm{Im}\,(V_{Ne}V^*_{N\mu}V_{n\mu}V^*_{ne})\frac{mg^4 r}{512\pi^3} \times$$

$$\int\limits_{q(r,z)}^{1+r^2} dx \int\limits_{z}^{y_+} dy_1 \mathrm{Im}\left[\frac{1}{1-y_1-r_R^2+i\gamma} \times \frac{1}{1-y_2-r_R^2-i\gamma}\right] g(x,y_1,r,r_R), \quad \text{(B.1)}$$

where

$$g(x,y_1,r,r_R) = [(1+r^2-x)(4+r^2 r_R^{-2}) - 2r_R^{-2}y_1(1-y_1-r_R^2) - 2r_R^{-2}y_2(1-y_2-r_R^2)] \quad \text{(B.2)}$$

$$y_{\pm} = \frac{1}{2}[2 - x \pm \sqrt{x^2-4r^2}] \quad \text{(B.3)}$$

$$q(r,z) = \frac{(1-z)^2+r^2}{1-z} \quad \text{(B.4)}$$

As a result of integration by y_1 one has:

$$\Gamma_{y_1>z} - \overline{\Gamma}_{y_2>z} = \mathrm{Im}\,(V_{Ne}V^*_{N\mu}V_{n\mu}V^*_{ne})\frac{rmg^4}{512\pi^3} \times$$

$$\int\limits_{q(r,z)}^{1+r^2} dx[a(r,r_R,z)c(r,r_R,z)] + b(r,r_R,z)] \quad \text{(B.5)}$$

$$a(r,r_R,z) = (1-x+r^2)(8+r^2r_R^{-2}-2r_R^{-2}x)-4(1-r_R^2)(1-r^2r_R^{-2})-4\gamma^2r_R^{-2} \tag{B.6}$$

$$c(r,r_R,z) = (\arctan\frac{y_+ + x - 1 - r_R^2}{\gamma} - \arctan\frac{z+x-1-r_R^2}{\gamma} - \arctan\frac{y_+ - 1 + r_R^2}{\gamma} + \arctan\frac{z-1+r_R^2}{\gamma})\frac{1}{x-2r_R^2} \tag{B.7}$$

$$b(r,r_R,z) = -2\gamma r_R^{-2}(ln((y_+ - 1 + r_R^2)^2+\gamma^2) - ln((z-1+r_R^2)^2+\gamma^2) - ln((y_+ + x - 1 + r_R^2)^2+\gamma^2) - ln((z+x-1+r_R^2)^2+\gamma^2)) \tag{B.8}$$

where $x = \frac{2\epsilon_n}{m}$, ϵ_n neutrino energy.

The witdh of the decay $N \to e^-\mu^+ n$ is:

$$\Gamma = \frac{mg^4}{512\pi^3}\left[|V_{Ne}|^2|V_{N\mu}|^2 + |V_{Ne}|^2|V_{n\mu}|^2\right]\int\limits_{2r}^{1+r^2}dx\int\limits_{y_-}^{y_+}dy_1\left[\frac{g(x,y_1,r,r_R)}{(1-y_1-r_R^2)^2+\gamma^2}\right] - \frac{rmg^4}{256\pi^3}\mathrm{Re}(V_{Ne}V^*_{N\mu}V_{n\mu}V^*_{ne})\int\limits_{2r}^{1+r^2}dx\int\limits_{y_-}^{y_+}dy_1\left[\frac{g(x,y_1,r,r_R)}{x-2r_R^2}\frac{1-y_1-r_R^2}{(1-y_1-r_R^2)^2+\gamma^2}\right] \tag{B.9}$$

The first term of formula (B9) is the sum of squares of amplitudes M_1, M_2 and in the limit of $\gamma \to 0$ (i.e. when $m - m_R \gg \Gamma_R$,$m_R - m_n \gg \Gamma_R$) will give the main contribution to the width:

$$\Gamma = \Gamma(N \to W_R^+ e^-)Br(W_R^+ \to n\mu^+) + \Gamma(N \to W_R^-\mu^+)Br(W_R^- \to ne^-) \tag{B.10}$$

The second term in the formula (B9) is the interference of amplitudes and for the range of masses $m - m_R \sim \Gamma_R$, $m - m_R \sim \Gamma_R$ one can not ignore it in general. However, even in this range interference can be considerably less than squares of amplitudes if the multiplier $r\mathrm{Re}(V_{Ne}V^*_{N\mu}V_{n\mu}V^*_{ne})$ in the interference term in (B9) is suffuciently less than $[|V_{Ne}|^2|V_{N\mu}|^2 + |V_{Ne}|^2|V_{n\mu}|^2]$ which contains the sum of the squared amplitudes of (B9).

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Figure Captions

Fig.1 Diagramms contributing to the Majorana nentrino decays $N \to e^- \mu^+ n (N \to e^+ \mu^- n)$.

Fig.2 Loop diagramms contributing to the Majorana nentrino decays $N \to e^- \mu^+ n (N \to e^+ \mu^- n)$.

Fig.3 Function $f(r, r_R, z)$ versus z at $r = 0.8$. Curves 1,2,3 denoted $r_R = o.95, 0, 9, 0, 85$ respectively.

Fig.4 Function $f(r, r_R, z)$ versus z at $r = 0.6$. Curves 1,2,3,4 denoted $r_R = 0.7, 0.9, 0.8$ respectively.

Fig.5 Function $f(r, r_R, z)$ versus z at $r = 0.3$. Curves 1,2,3,4 denoted $r_R = 0.9, 0.5, 0.8, 0.7$ respectively.